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Fifty years of research at the Landau Institute for Theoretical Physics (on the 100th anniversary of the birth of I M Khalatnikov)

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Abstract. Reviewing all the basic research performed at the Landau Institute for Theoretical Physics, Russian Academy of Sciences that has made a significant contribution to physics is an unrealistic task. Therefore, the discussion is restricted to only those studies that have directly affected the author's explorations for 50 years (1968–2018). I M Khalatnikov created a unique institution that brought together virtually all areas of theoretical physics of importance, thus opening vast opportunities for scientific collaboration. The Landau Institute's multidisciplinary environment was a significant driver of research.

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1. Khalatnikov lectures and emergent relativity in superfluids

The Khalatnikov lectures on superfluid ^{4}He for students at the Kapitza Institute led me to the strange connection between superfluid hydrodynamics and special and general relativity (probably in 1967). The relativistic character of the flow of superfluid ^{4}He is manifested at low temperatures, where the normal component is represented by 'relativistic' excitations with a linear spectrum — phonons. Equation (3.13) in Khalatnikov's book [1] shows the free energy of phonons in the presence of the counterflow $w = v_n - v_s$ (the flow of the normal component of a liquid with respect to the superfluid component):

$$F_{\rm ph}(T, w) = \frac{F_{\rm ph}(T)}{(1 - w^2/c^2)^2} \sim \left(\frac{T}{\sqrt{1 - w^2/c^2}}\right)^4, \qquad (1)$$

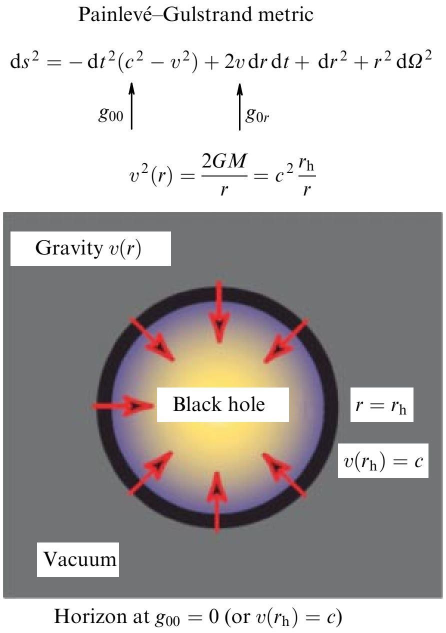

Figure 1. (Color online.) Black hole in the Painlevé–Gullstrand metric.

with $F_{ph}(T) \propto T^4$. This equation recalls the Tolman law in general relativity, $T(r) = T/(g_{00}(r))^{1/2}$, which may suggest (and indeed suggested for me) the idea that the vacuum is superfluid; matter is represented by excitations; the gravitational field is the result of the flow of the vacuum with superfluid velocity $v_s^2/2 = GM/r$ (in full thermodynamic equilibrium one has $v_n = 0$, while v_s may depend on the coordinates, and $g_{00}(r) = 1 - w^2/c^2 = 1 - v_s^2(r)/c^2$); and GR is some extension of the Landau–Khalatnikov two-fluid hydrodynamics. My attempt to share this crazy idea with Sharvin, who governed the student seminar at the Kapitza Institute, was not successful. I got the response that GR is a very beautiful theory, and it should not be spoiled by unjustified models. Now I fully agree with his absolutely correct response.

However, later on, the papers by Unruh on black hole analogs in moving liquids appeared [2, 3]. The effective metric experienced by sound waves in liquids (or correspondingly phonons in superfluids) became known as the acoustic metric, and the flow of a liquid with the acoustic horizon as the river model of black holes [4]. In GR, the flow metric with the acoustic horizon corresponds to the Painlevé–Gullstrand (PG) metric [5, 6] (Fig. 1). Following this idea, Ted Jacobson and I considered the Painlevé–Gullstrand metric, which emerges for fermionic excitations in superfluid ^{3}He — Bogoliubov–Nambu quasiparticles — and discussed the possibility of creating analogs of horizons of black holes and white holes using moving textures, solitons [7].

The PG metric is useful for considering the processes inside and across the horizon, such as Hawking radiation. Both for a real black hole and for its condensed matter analog, Hawking radiation can be considered as semiclassical quantum tunneling across the horizon (see Ref. [8] for the black hole analog and Ref. [9] for the real black hole).

The acoustic metric allows us to simulate many different spacetimes. I remember the talk by Polyakov in 1981, where he mentioned that the Minkowski signature in GR may emerge from the Euclidean one in a kind of symmetry-breaking phase transition. The effective Minkowski-to-Euclidean signature change can be probed in particular using the acoustic metric for the Nambu–Goldstone mode in the Bose–Einstein condensate of magnons [10, 11].

In 2003, many different analogies between condensed matter, on the one hand, and relativistic quantum fields and gravity, on the other hand, have been collected in book [12]. In the fermionic Weyl and Dirac materials, emergent gravity is formulated in terms of tetrad fields (see also the recent papers [13, 14]), instead of the metric gravity emerging in bosonic condensed matter systems. Moreover, in Weyl materials, gravity emerges together with all the ingredients of the relativistic quantum field theories (relativistic spin, chiral fermions, gauge fields, Γ-matrices, etc.) (see also Section 15).

2. Iordanskii and macroscopic quantum tunneling

My supervisor at the Landau Institute was Iordanskii — the author of thermal nucleation of vortices [15] and, together with Finkelshtein, of the quantum formation of nucleation centers in a metastable crystal [16, 17]. Iordanskii suggested to me the problem of quantum nucleation of vortices in superfluids. This resulted in a paper on vortex nucleation in moving superfluids by quantum tunneling [18].

The main difference from other types of macroscopic quantum tunneling is that the role of the canonically conjugate quantum variables is played by the z and r coordinates of the vortex ring. This provides the volume law for the vortex instanton: the action contains the topological term $S_{top} = 2\pi\hbar n V_L = 2\pi\hbar N_L$, where n is the particle density; V_L is the volume inside the surface swept by the vortex line between its nucleation and annihilation; and N_L is the number of atoms inside this volume (see Section 26.4.3 in book [12]). For other linear topological defects and for fundamental strings, the area law is applied [19].

The further development of this macroscopic quantum tunneling in superconductors can be found in review paper [20], where most of the authors are from the Landau Institute: Feigel'man, Geshkenbein, and Larkin.

3. Khalatnikov, superfluid ^{3}He, paradox of angular momentum, and the chiral anomaly

It is not surprizing that the epoch of superfluid ^{3}He at the Landau Institute was initiated by Khalatnikov. My participation in this programme started with collaboration with Khalatnikov and Mineev on the extension of the Landau–Khalatnikov hydrodynamics of superfluid ^{4}He to the dynamics of a mixture of Bose and Fermi superfluids [21]. The most interesting topic there was the Andreev–Bashkin effect, when the superfluid current of one component also depends on the superfluid velocity of the other component [22]. The seminal paper by Andreev and Bashkin [22] had just been published in the previous issue of *JETP*, which demonstrated the traditionally close relations between the Landau and Kapitza Institutes.

The first attempts to extend the Landau–Khalatnikov hydrodynamics to the hydrodynamics of the chiral superfluid ^{3}He-A immediately showed some strange paradox related to the intrinsic angular momentum of the liquid with Cooper pairing into the $p + ip$ state [23–25]. The calculated magnitude of the dynamical angular momentum was smaller by the factor $(\Delta_0/E_F)^2$ than the expected angular momentum of the stationary state, $L_z = \hbar N/2$, which corresponds to $\hbar$ for each of $N/2$ Cooper pairs, where N is the number of atoms. With Δ_0 being the gap amplitude in the fermionic quasiparticle spectrum and E_F the Fermi energy, this factor is very small, $(\Delta_0/E_F)^2 \sim 10^{-5}$.

Only about ten years later, some understanding was reached [26, 27] that the source of the angular momentum paradox and of the other related paradoxes in the ^{3}He-A dynamics was the analog of the chiral anomaly in RQFT [see Eqn (7)]. The chiral anomaly is realized in a quantum vacuum with Weyl fermions, and the topologically protected Weyl fermions emerge in the chiral superfluid ^{3}He-A [12]. The Khalatnikov–Lebedev hydrodynamics of chiral superfluid ^{3}He [28, 29] had to be modified to include the chiral anomaly effects [27].

The effect of the chiral anomaly has been experimentally verified in the dynamics of skyrmions in ^{3}He-A [30] (see Section 20; see also the recent discussion on the connection between the chiral anomaly and the angular momentum paradox in chiral superfluids and superconductors [31, 32]).

4. Polyakov monopole and vortex with a free end

In 1974 Polyakov gave a talk at the Landau Institute seminar in Chernogolovka on the hedgehog-monopole in the Higgs field, which later got the name 't Hooft–Polyakov monopole [33, 34]. Inspired by this talk, Mineev and I suggested the analog of magnetic monopole in ^{3}He-A [35] (the same suggestion was made by Blaha [36]). As distinct from the 't Hooft–Polyakov monopole, this monopole terminates the linear defects — vortices and strings. It can terminate either a singular doubly quantized vortex with $N = 2$ (Fig. 2a), which we called a vortex with a free end, two singly quantized vortices with $N = 1$ (Fig. 2b), or four half-quantum vortices with $N = 1/2$ (Fig. 2c).

In electroweak theory, such a monopole terminating the electroweak string is known as the Nambu monopole [37]. The condensed matter analog of the magnetic monopole, which terminates the string, has been observed in cold gases [38].

5. Ferromagnetic hedgehog as a magnetic monopole in a synthetic field

The monopole-hedgehog topic started by Polyakov and also some vague ideas about the possible emergence of gauge fields developed as follows. It appeared that, in ferromagnets, the Berry phase gives rise to a synthetic electromagnetic field [39]:

$$F_{ik} = \partial_i A_k - \partial_k A_i = -\frac{1}{2}\mathbf{m}\left(\partial_i \mathbf{m} \times \partial_k \mathbf{m}\right), \tag{2}$$

$$E_i = -\partial_t A_i - \partial_i A_0 = \frac{1}{2}\mathbf{m}\left(\partial_t \mathbf{m} \times \partial_i \mathbf{m}\right), \tag{3}$$

where $\mathbf{m}$ is the unit vector of magnetization. The effective electric and magnetic fields are physical: they act on electrons in ferromagnets and produce, in particular, the spin-motive force, induced by time and space derivatives of magnetization [39, 40]. This force is proportional to $\mathbf{E} - \mathbf{v} \times \mathbf{B}$, similarly to that in quantum electrodynamics. The spin-motive force is enhanced in the presence of the Dzyaloshinskii–Moriya interaction [41].

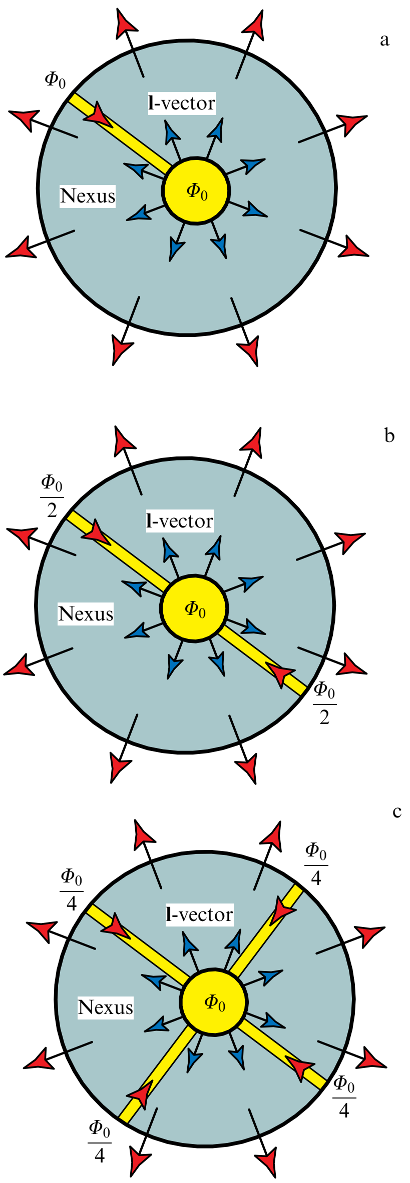

Figure 2. (Color online.) Magnetic monopole in chiral superconductor – the analog of the Nambu monopole. (a) Monopole, which terminates the doubly quantized vortex, $N = 2$. (b) The same monopole terminates two vortices with $N = 1$. (c) Nexus — monopole with four half-quantum ($N = 1/2$) vortices — the Alice strings in Fig. 4. Red arrows show direction of magnetic flux, which is brought to the monopole by vortices, and then radially propagates from the monopole. The blue arrows are the field of the orbital vector $\mathbf{l}$, which forms the hedgehog.

The hedgehog in ferromagnets in Fig. 3a appeared to be the monopole in the Berry phase magnetic field $\mathbf{B}$ [39]. This somewhat recalls the Polyakov hedgehog in the Higgs field, which at the same time represents the magnetic monopole. However, as distinct from the Weyl material scenario of the emergent gauge field, in this Berry phase scenario, full analogy with the electromagnetic field is missing.

6. Novikov, topology, Alice string, Berezinzkii

During his seminar talk at the Landau Institute on the hedgehog-monopole, Polyakov mentioned that mathematicians claim that the hedgehog cannot be destroyed for topological reasons. This led to the intensive study of

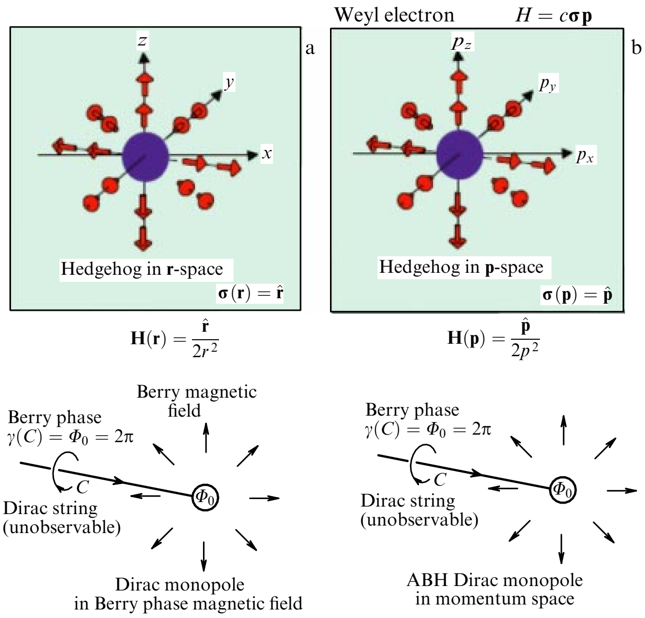

Figure 3. Berry phase magnetic monopoles in real and momentum space. (a) Berry phase magnetic monopole in ferromagnets. (b) Abrikosov–Beneslavskii–Herring monopole in momentum space, which is responsible for the topological stability of Weyl fermions. Both have an unobservable Dirac string with 2π winding of the Berry phase.

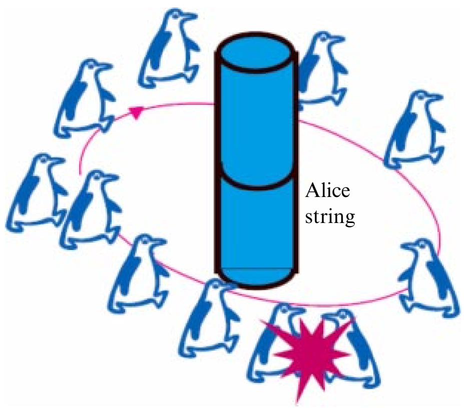

Figure 4. Half-quantum vortex as an Alice string [45]. Matter continuously transforms into antimatter after circling the Alice string. Two penguins start to move in opposite directions around the string. When they meet each other, they annihilate.

topology in physics and discussions with the members of the Novikov group at the Landau Institute (Bogoyavlensky, Grinevich, and others). In addition, the Anisimov–Dzyaloshinskii paper on disclinations appeared [42], where the variety of structures in liquid crystals was discussed. Mineev and I wanted to understand how and why these and other structures — including our vortex with a free end (an analog of the Nambu monopole) — were topologically stable or not.

This led us to the homotopy group classification of topological structures [43, 44]. Among these structures, some unexpected exotic topological objects were suggested, such as the half-quantum vortex in ^{3}He-A [43]. In RQFT, the analog of the half-quantum vortex is the Alice string (Fig. 4) discussed by Schwarz [45], who collaborated with Belavin and

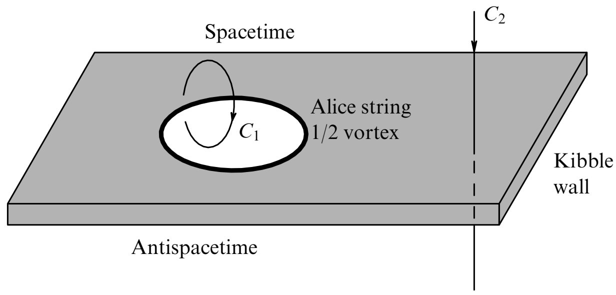

Figure 5. In ^{3}He-B, the Alice string (the half-quantum vortex) becomes the termination line of the nontopological domain wall — the Kibble wall [48]. There are two paths to antispacetime: the safe route around the Alice string (along the contour C_1) and the dangerous route along C_2 across the Kibble wall [50].

Polyakov on the instanton problem [46]. Experimentally, the half-quantum vortices were observed only 40 years later, first in the time-reversal symmetric polar phase of ^{3}He [47], and finally in chiral ^{3}He-A [48]. Moreover, it was found that they survive the phase transition to ^{3}He-B, where the half-quantum vortex is topologically unstable: it becomes the termination line of the nontopological domain wall — an analog of the Kibble cosmic wall [49] (Fig. 5).

Vortices in ^{3}He-A are described by the Z_4 homotopy group, which means that $1/2 + 1/2 + 1/2 + 1/2 = 1 + 1 = 2 = 0$, and thus 4 half-quantum vortices can terminate at the monopole shown in Fig. 2. Furthermore, in systems such as liquid crystals, the topological defects — disclinations — may obey even non-Abelian homotopy groups. All this piqued Berezinzkii's interest in the possibility of extending the BKT transition [51–53] to more general symmetry breaking patterns, but, unfortunately, he passed away in 1980.

Novikov himself also participated in the ^{3}He work. In particular, he resolved the paradox related to the number of Nambu–Goldstone (NG) modes in ^{3}He-A: in the weak coupling limit, there are 9 NG modes, but only 8 broken symmetry generators [54, 55]. Novikov formulated a new counting rule [56]: the number of NG modes coincides with the dimension of the tangent space. The mismatch between the total number of NG bosons and the number of broken symmetry generators equals the number of extra flat directions in the Higgs potential.

7. Novikov, topology, skyrmions

Our next step in the classification of topological structures in condensed matter was triggered again by the Landau Institute environment: the Belavin–Polyakov topological object in 2D Heisenberg ferromagnets [57], dynamical solitons discussed by the Zakharov group [58], and discussions with members of the Novikov group (Golo and Monastyrsky [59]). All this led us to the classification of continuous structures in terms of relative homotopy groups [60].

These structures include, in particular, analogs of 3D skyrmions: the particle-like solitons described by the π_3 homotopy group [61] (which are called the Shankar monopole [62]). Isolated 3D skyrmions have been observed in cold gases [63]. In superfluid ^{3}He, it is still difficult to stabilize the isolated skyrmions, but π_3 objects at the crossing of 1D and 2D topological solitons have been created experimentally [64]; see the topological analysis of these combined objects by Makhlin and Misirpashaev [65].

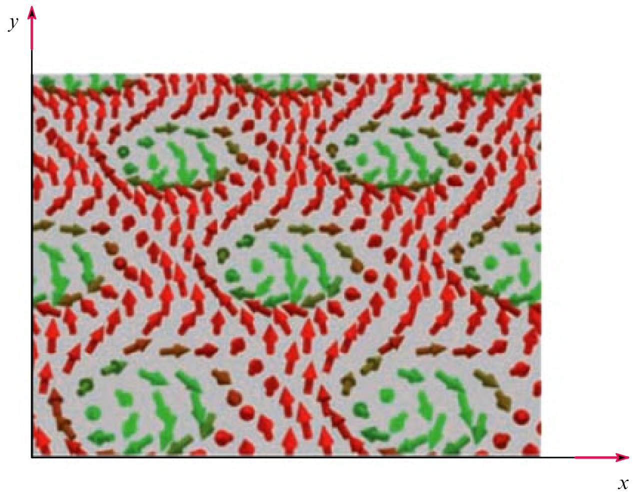

Figure 6. Skyrmion lattice in rotating chiral superfluid. Each cell has two quanta of circulation of superfluid velocity.

8. Kopnin and the vortex skyrmion lattice

In contrast to 3D skyrmions, 2D skyrmions are typical in the Helsinki experiments with the chiral ^{3}He-A in a rotating cryostat. In superfluid ^{3}He-A, the vorticity can be continuous (nonsingular) and can form a periodic texture in the rotating vessel — the 2D skyrmion lattice, which was discussed with Kopnin [66] (Fig. 6). This paper opened the collaboration with Kopnin. The 2D skyrmions were later identified in NMR experiments [67]. Subsequently, the change in the topological charge of the skyrmion was observed in ultrasound experiments [68]. Sweeping the magnetic field, we could see the first-order topological transition between different configurations. In small fields, the skyrmion has nontrivial charges, both in the orbital and in the spin vector fields, $N_l = 1$ and $N_d = 1$. In strong fields, the skyrmion loses one of the winding numbers, $N_l = 1$ and $N_d = 0$.

A similar skyrmion lattice was suggested by Kopnin for anisotropic superconductivity, in which the symmetry breaking pattern is $SU(2)_S \times U(1)_N/Z_2 \to U(1)_{S_z - N/2}$ [69].

9. Bekarevich–Khalatnikov theory of rotating superfluid

Figure 7 demonstrates a skyrmion lattice in a rotating cryostat in the presence of an interface between ^{3}He-A and ^{3}He-B [70]. Experimentally, one can produce a different pattern of rotating superfluids. In particular, one of the superfluids, the A-phase, contains an equilibrium number of vortices, while the other one, the B-phase, is vortex-free (Fig. 7b). In this case, the vortices in A-phase bend and form a vortex sheet. To describe this bending, we used the hydrodynamic equations derived by Bekarevich and Khalatnikov [71].

10. Interface instability: Korshunov, Kuznetsov, Lushnikov

The arrangement in Fig. 7b allowed us to study experimentally an analog of the Kelvin–Helmholtz instability in superfluids [72]. At some critical velocity of rotation, the interface becomes unstable to the formation of ripplons at the interface

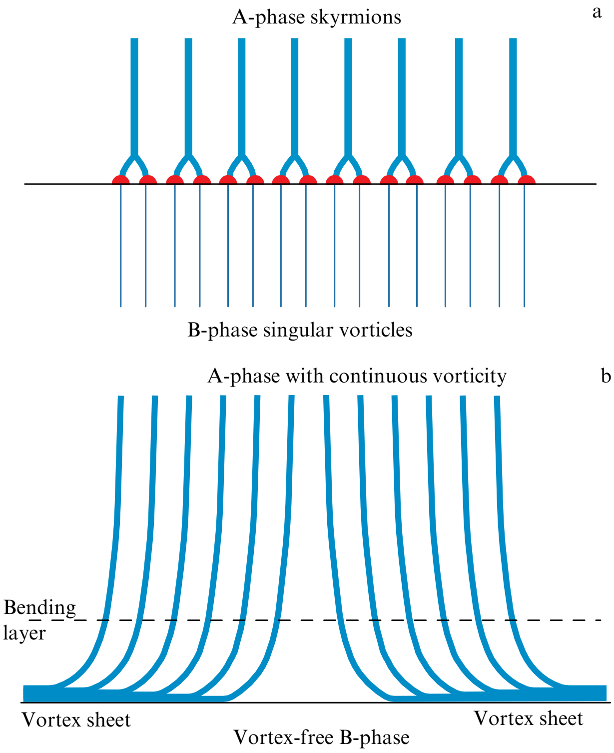

Figure 7. Interface between chiral superfluid ^{3}He-A and non-chiral ^{3}He-B in a rotating cryostat. (a) Skyrmion lattice in the A-phase transforms into a conventional vortex lattice in the B-phase. The end point of the B-phase vortex is an analog of the Nambu monopole. (b) Skyrmion lattice transforms into the vortex sheet at the interface, while the B-phase is made vortex free. The bending of vorticity at the interface obeys the Bekarevich–Khalatnikov theory of rotating superfluids [71].

[73]. Originally, the Kelvin–Helmholtz (KH) instability in superfluids was studied by Korshunov [74, 75]. Instead of the conventional KH instability of the interface between two fluids, Korshunov considered a rather unusual case: using the Landau–Khalatnikov two-fluid model, he studied the instability of the surface of the liquid under counterflow of the superfluid and normal components of the same liquid. It happens that the arrangement in Fig. 7b is strongly resembles the Korshunov case. On one side of the interface, the vortex-full A-phase rotates together with the container, $\langle \mathbf{v}_{sA} \rangle = \mathbf{v}_{nA} = \boldsymbol{\Omega} \times \mathbf{r}$. On the other side of the interface, in the vortex-free B-phase, only the normal component rotates with the container, while its superfluid component is at rest: $\mathbf{v}_{sB} = 0$, $\mathbf{v}_{nB} = \boldsymbol{\Omega} \times \mathbf{r}$. That is why the instability occurs due to the counterflow on the B-phase side, $\mathbf{w}_B = \mathbf{v}_{nB} - \mathbf{v}_{sB}$.

The nonlinear stage of the KH instability was considered by Kuznetsov and Lushnikov [76, 77]. In our experiments, the development of interface instability leads to penetration of the A-phase skyrmions through the interface to the B-phase, where they are finally transformed into singular vortices. Figure 8 demonstrates the possible scenario of this transformation. However, a complete analysis of the this process is still lacking.

11. Vortex creation in a micro-Big Bang, Kopnin, Kamensky, Manakov

Another nonlinear out-of-equilibrium phenomenon studied experimentally in superfluid ^{3}He had its origin in the interplay of high-energy physics and cosmology. This is the nucleation

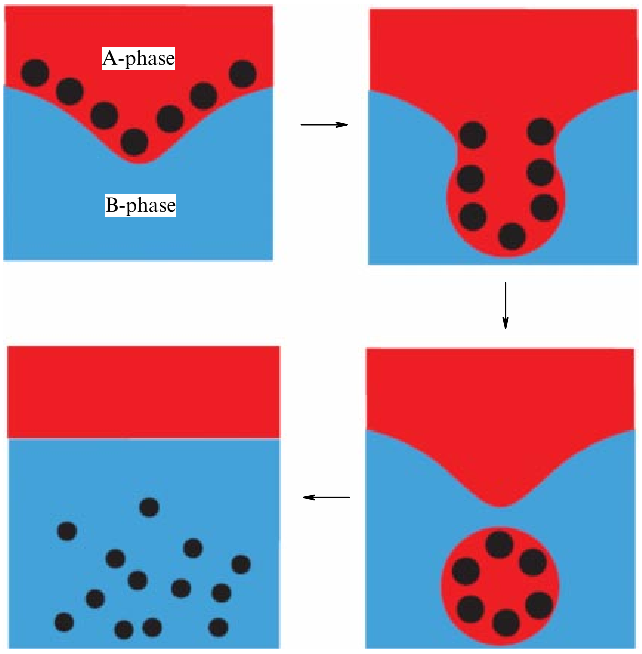

Figure 8. Development of the Kelvin–Helmholtz type of instability at the interface between the vortex-full chiral superfluid ^{3}He-A and vortex-free nonchiral ^{3}He-B in the rotating cryostat in Fig. 7b. The instability leads to the creation of vortices in ^{3}He-B. A possible scenario: the droplet of the A-phase with vorticity concentrated in the skyrmions penetrates through the AB interface, where the vorticity transforms into singular vortices. NMR experiments show that the number of B-phase vortices formed after instability is consistent with the wavelength of the critical ripplon.

of topological defects during the phase transition in the expanding universe [78], named the Kibble–Zurek mechanism of defect formation. In ^{3}He-B, the Big Bang event was simulated by neutron irradiation, which caused a nuclear reaction and heating of the bubble about 100 μm in size above the superfluid phase transition temperature [78] (Fig. 9). Then, the cooling of the bubble back through the second order phase transition to the broken symmetry state resulted in the creation of vortices measured in NMR experiments.

The explanation of the observed vortex creation in the framework of the Kibble–Zurek scenario looks reasonable. Moreover, it is supported by the correct power-law dependence of the number of the created vortices on the velocity of the superfluid. Nevertheless, modifications and extensions of the Kibble–Zurek scenario were necessary in order to take into consideration the inhomogeneity of the process. In particular, the effect of propagation of the transition front was considered in the paper by Kibble and me [80] and in papers by Kopnin et al. [81–83]. But the original idea that vortices can be created by a propagating front of the second-order phase transition belongs to Kamensky and Manakov [84].

12. Instantons, Belavin–Polyakov–Schwartz–Tyupkin, chiral superfluids, superconductors

Polyakov [85] and Belavin–Polyakov–Schwartz–Tyupkin (BPST) [46] instantons inspired the study of instanton structures in condensed matter. The $1+1$ instanton lattice [86] in Fig. 10 serves to explain the oscillations observed in counterflow experiments in chiral superfluid ^{3}He-A [87]. This $1+1$ lattice in the (z, t) plane, where z is the coordinate along the counterflow, is the counterpart of the $2+0$ skyrmion lattice in Fig. 6. A similar $1+1$ instanton structure, but in terms of the (z, t) counterparts of the Abrikosov vortex lattice, is discussed for superconductors by Ivlev and Kopnin [88].

13. Dzyaloshinskii, spin glasses. General hydrodynamics and dimensionless physics

Magnetic materials are natural subjects for studies of topologically stable structures, the main expert at Landau Institute having been Dzyaloshinskii — the author of the Dzyaloshinskii–Moriya interaction [89, 90]. The common interest in topological defects in magnetic materials led to our collaboration. We considered frustrations in spin glasses introduced by Villain [91] and suggested that, on the macroscopic hydrodynamic level, the frustrations can be described in terms of topological defects — disclinations — which destroy the long range magnetic order [92]. The continuous distribution of disclinations and their dynamics can be described using effective gauge fields: the U(1) gauge field in XY spin glasses, and SU(2) gauge field in Heisenberg spin glasses. This provides another scenario for emergent gauge fields, in addition to the Berry phase scenario in ferromagnets and the Weyl point scenario in ^{3}He-A and in Weyl semimetals.

The hydrodynamics of systems with distributed defects was then extended to superfluids with vortices and crystals with dislocations and disclinations [93, 94]. The relevant gauge fields which describe the distributed dislocations and disclinations are correspondingly the torsion and Riemann curvature in the formalism of general relativity.

It is important that the elasticity tetrads $E^a_\mu(x)$, which describe elastic deformations of a crystal lattice, can be expressed in terms of a system of deformed crystallographic coordinate planes, surfaces of constant phase $X^a(x) = 2\pi n^a$ [92, 95, 96]. The tetrads $E^a_\mu(x) = \partial_\mu X^a(x)$ have the dimension of inverse length (or inverse time). When these elasticity tetrads are applied to general relativity (the so-called superplastic vacuum [97]), one discovers that the Ricci curvature scalar R, the gravitational Newton constant G, and the cosmological constant Λ become dimensionless, $[G] = [R] = [\Lambda] = 1$. The higher order gravitational terms, such as R^2 and $R^{\mu\nu}R_{\mu\nu}$, are also dimensionless.

Moreover, all the physical quantities become dimensionless. The reason for this is that, in a superplastic vacuum, which can be arbitrarily deformed, the equilibrium size of the elementary cell is absent, and thus the microscopic length scale (such as the Planck scale) is absent (Fig. 11). That is why all the distances are measured in terms of the integer positions of the nodes in the crystal.

The hydrodynamics of systems with topological defects can be useful for describing different types of spin and orbital glasses observed in the superfluid phases of ^{3}He in aerogel [98] (see also Section 19).

14. Abrikosov–Beneslavskii–Herring monopole

The Polyakov hedgehog-monopole-instanton saga had one more important development, now extended to momentum space. The momentum-space counterpart of the Berry phase magnetic monopole is shown in Fig. 3b. This figure shows the topological signature of the 2×2 Hamiltonian describing

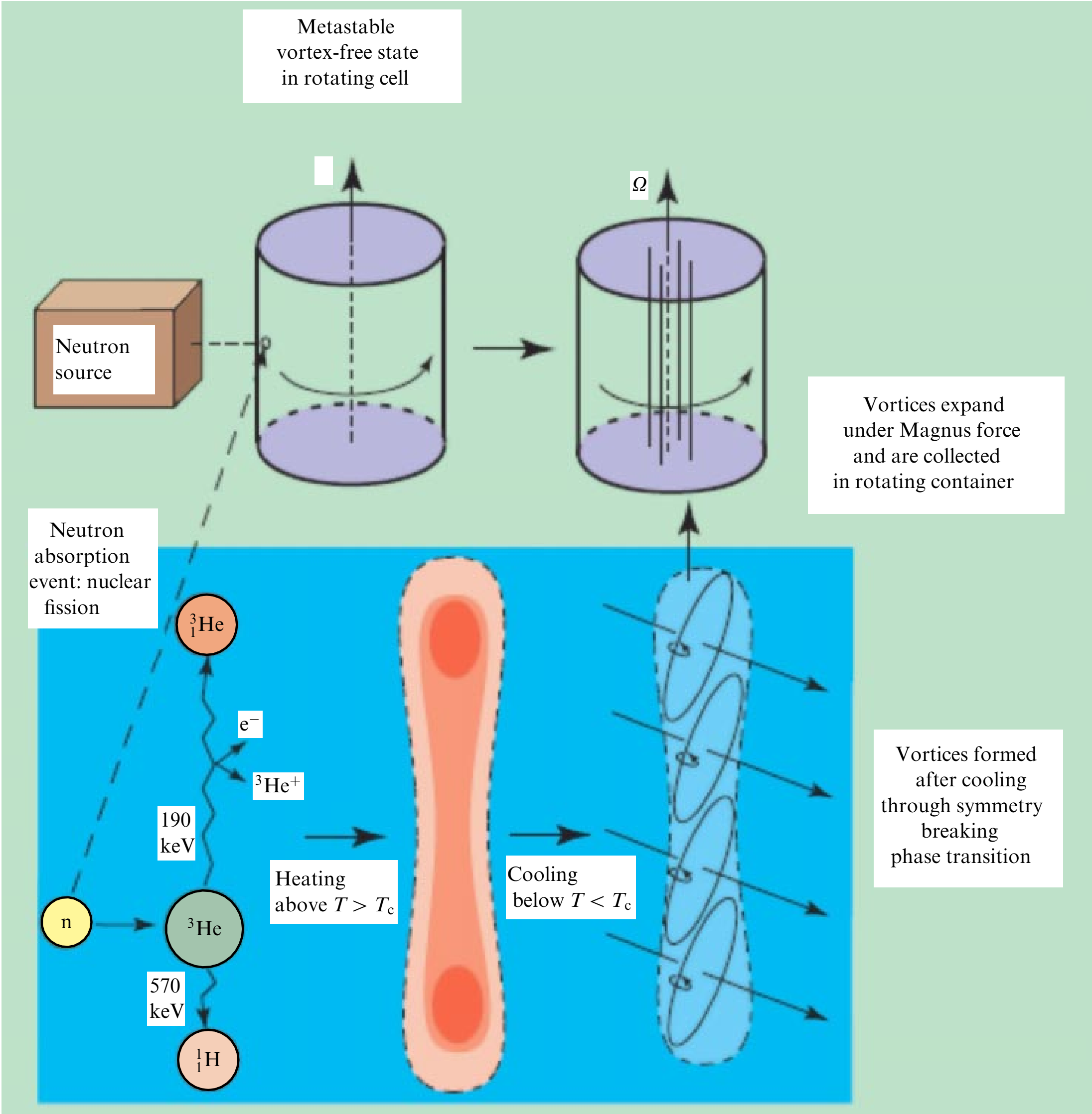

Figure 9. Vortex formation in a micro-Big Bang event caused by neutron irradiation [79].

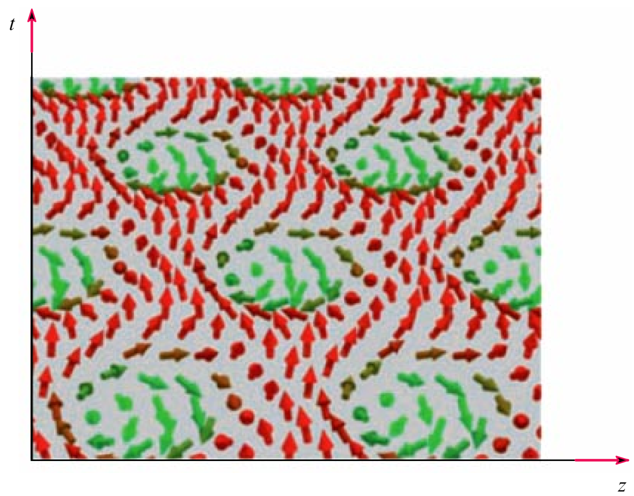

Figure 10. Instanton lattice in the dynamics of a chiral superfluid. It looks like the skyrmion lattice in Fig. 6, but in the (z, t)-plane.

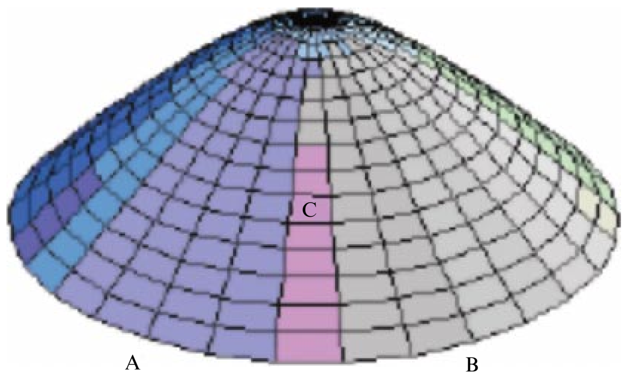

Figure 11. 2D illustration of the $3 + 1$ superplastic vacuum, in which the equilibrium size of the elementary cell is absent.

Weyl fermions [99]. The spin (or pseudospin in Weyl materials) forms the hedgehog in momentum space, representing the Berry phase monopole in momentum space. The stability of this hedgehog is also supported by the topology, but now it is the topology in momentum space. A topological description of the band contact points can be found in the paper by Novikov [100]. Topological stability of the Weyl point ensures the emergence of relativistic Weyl fermions in the vicinity of the hedgehog, even in a condensed matter nonrelativistic vacuum, such as in superfluid ^{3}He-A. Together with chiral Weyl fermions, gravity in terms of the tetrad fields and relativistic quantum gauge fields emerge in this superfluid. In other words, the whole Universe (or actually its caricature) can be found in a droplet of ^{3}He [12].

Unfortunately, at that time I was unaware of another, substantially older, Universe created by Abrikosov, who along with Beneslavskii considered relativistic Weyl fermions in semimetals [101–103]. But in 1998, after the Abrikosov-70 workshop in Argonne, I received from Abrikosov a reference to his papers. I then realized that as a student I had visited his seminar talk in Chernogolovka in 1970, where he discussed the 'relativistic' conical spectrum of electrons in semimetals. Though I later forgot about that seminar, it was somehow deep in my subconscious mind. The Berry phase hedgehog-monopole in momentum space could be called the Abrikosov–Beneslavskii–Herring (ABH) monopole.

The topological invariant which describes the Weyl node can also be written in terms of the Green's function with an imaginary frequency [104]. The Green's function has a point singularity in the 4D momentum space (ω, p_x, p_y, p_z) (see Eqn (6) in Section 18). It is the momentum-space analog of the instanton. A description in terms of the Green's function is important in the case of strong interaction, when the single-particle Hamiltonian is ill defined.

15. Gribov: relativistic quantum field theories in Majorana–Weyl superfluid ^{3}He-A, Moscow zero, quark confinement

Over the years, Gribov's help was extremely important to me. Although he was usually very critical during seminars at the Landau Institute, he patiently answered my questions, perhaps because I did not belong to the high-energy community, and sometimes he even shared his ideas with me. Gribov clarified to me various issues related to emergent relativistic quantum field theories (RQFTs) in the Weyl superfluid ^{3}He-A. In one case, I was puzzled by the logarithmically diverging term in the action for ^{3}He, which in terms of the effective U(1) gauge field has the form $(B^2 - E^2)\ln[E_{\rm UV}^4/(B^2 - E^2)]$, where $E_{\rm UV}$ is the ultraviolet cut-off (see Section 20 for a definition of the synthetic gauge field below Eqn (7)). This term becomes imaginary for $E^2 > B^2$, so what to do with that? From the discussion with Gribov, it became clear that this is nothing but the vacuum instability leading to Schwinger pair production, which occurs when the synthetic electric field exceeds the synthetic magnetic field.

Logarithmic divergence is the condensed matter analog of the zero charge effect — the famous 'Moscow zero' of Abrikosov, Khalatnikov, and Landau [105–107]. The zero charge effect is natural for the U(1) gauge field. However, to my surprise, the synthetic SU(2) gauge field, which emerges in ^{3}He-A too, also obeys the zero charge behavior instead of the expected asymptotic freedom found by Gross, Wilczek, and Politzer [108, 109]. The discussion with Gribov clarified this issue too. He simply asked me the question: "How many bosons and fermions do you have in your helium?" It appeared that the number of fermionic species is small compared to the number of bosonic fields. However, the fermions dominate because of the much larger ultraviolet cut-off $E_{\rm UV}$ and they lead to a zero charge.

Gribov also explained to me the origin of the Wess–Zumino term in the hydrodynamic action and some other things. All this resulted in the paper on anomalies in ^{3}He-A [110] with acknowledgment to Gribov for numerous and helpful discussions. The term in Eqn (4.9) there appeared after Gribov shared with me his view of the problem of the zero charge effect for massless fermions, and then I realized that the same term with the same prefactor exists in the superfluid hydrodynamics of ^{3}He-A. It is Eqn (25) in the Gribov paper [111].

Gribov was also the first to tell me that Bogoliubov quasiparticles in ^{3}He-A are Majorana fermions. The zero energy modes found by Kopnin in the core of the ^{3}He-A vortex [112] (see also [113] and Section 21) appeared to be Majoranas. Later, I checked that the zero energy level does not shift from zero, even in the presence of impurities [114], which is an important signature of the Majorana nature of the mode.

I continued the discussions with Gribov, even after he moved to Hungary, in particular in relation to quark confinement. My proposal to explain the confinement in terms of the ferromagnetic quantum vacuum after discussions with Gribov appeared to be a modification of the idea of the Savvidy vacuum [115].

More recently, Klinkhamer and I tried to extend the Gribov picture of confinement in QCD as diverging mass at low k and to apply it to cosmology [116]. We came to the following estimation for the vacuum energy density (cosmological constant Λ) in the present epoch [117]:

$$\Lambda = \rho_{\rm vac} \sim H \Lambda_{\rm QCD}^3 \,, \tag{4}$$

where H is the Hubble parameter, and $\Lambda_{\rm QCD}$ is the QCD energy scale. Unfortunately, Gribov's helpful criticism is now lacking (he passed away in 1997). The linear dependence on H obtained in his phenomenological theory of confinement has also been proposed in other approaches to QCD [118, 119].

For the de-Sitter universe, one has $\Lambda \sim H^2 E_{\rm Planck}^2$, which gives

$$\Lambda \sim \frac{\Lambda_{\rm QCD}^6}{E_{\rm Planck}^2} \,. \tag{5}$$

On the other hand, since $\Lambda_{\rm QCD}$ is on the order of proton mass $m_{\rm p}$, equation (5) corresponds to the early suggestion by Zeldovich, $\Lambda \sim G m_{\rm p}^6$ [120] (see also the recent paper by Kamenshchik, Starobinsky, and co-authors on the Pauli–Zeldovich mechanism of cancellation of vacuum energy divergences [121]).

16. Gor'kov: unconventional superconductivity, Weyl points, and Dirac lines

Our collaboration with Gor'kov started when he returned from a conference, where new heavy-fermion superconductors had been discussed. Our collaboration led to the symmetry classification of superconducting states [122, 123]. Most of the unconventional superconducting states have nodes in the energy spectrum: Weyl points, Dirac points, and Dirac nodal lines. One of the configurations with 8 Weyl points (4 right and 4 left) is shown in Fig. 12. The extension of this configuration to 4D space produces the 4D analog of graphene [124, 125], with 8 left and 8 right Weyl fermions, as in each generation of Standard Model fermions (see also the

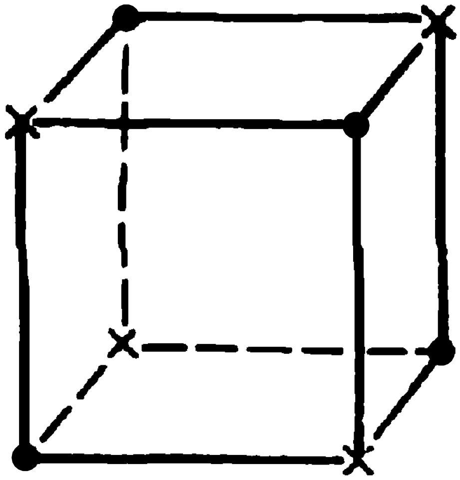

Figure 12. Arrangement of nodes in an energy spectrum in superconductors of class $O(D_2)$. The points denote four Weyl nodes with topological charge $N = +1$ (the winding number of the hedgehog with spins outwards), and crosses denote four Weyl nodes with $N = -1$ (the winding number of the hedgehog with spins inwards). In the vicinity of each Weyl node with $N = +1$, chiral right-handed Weyl fermions emerge, while $N = -1$ is the topological charge of the left-handed quasiparticles. This arrangement of Weyl nodes can be compared with 8 right-handed and 8 left-handed particles (quarks and leptons) in each generation of Standard Model fermions. (Figure from Ref. [123].)

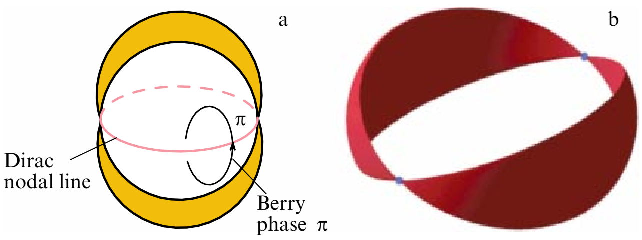

Figure 13. Nodal line in the polar phase of ^{3}He (a) and its transformation into a Bogoliubov Fermi surface under superflow (b).

paper in memory of Gor'kov [126], where the superconducting state with 48 Weyl fermions is discussed).

Dirac lines also appear in many classes of superconductivity, including cuprate superconductors. The Dirac line has an important effect on the thermodynamics of superconductors [127]. The reason is that, in the presence of a supercurrent, the nodes in the spectrum transform into Fermi surfaces (Fig. 13). Such Fermi surfaces emerge in superconductors due to broken time reversal symmetry or parity, and now they are called Bogoliubov Fermi surfaces [128]. The Bogoliubov Fermi surface ensures a nonzero density of states (DoS), which in the case of the nodal line is proportional to the superfluid velocity. For the Abrikosov vortex lattice, the flow around vortices produces a DoS which is proportional to $\sqrt{B}$ [127–129]. Gor'kov called this phenomenon 'koreshok' — the diminutive form of the Russian word 'koren' (root).

17. Paper by Dzyaloshinskii, Polyakov, and Wiegmann and θ-term in ^{3}He-A

The paper by Dzyaloshinskii, Polyakov, and Wiegmann [130] inspired work on the possible θ-term in thin films of chiral superfluid ^{3}He-A [131]. The consequences of that are the intrinsic quantum Hall effect, spin quantum Hall effect, and exotic spin and statistics of solitons, which depend on film thickness [132]. These studies were done while conducting extremely useful discussions with Wiegmann.

18. Yakovenko, Grinevich, and topology in momentum space

During our collaboration with Yakovenko, we expressed the intrinsic quantum Hall and spin quantum Hall effects via π_3 topological Chern numbers in terms of the Green's function [133]. The same invariants, but where the integral is around the Weyl point in the 4D p_μ space,

$$N = \frac{1}{24\pi^2}\, e_{\mu\nu\lambda\gamma}\, \mathrm{tr} \int_\sigma \mathrm{d}S^\gamma \mathcal{G}\partial_{p_\mu} \mathcal{G}^{-1} \mathcal{G}\partial_{p_\nu} \mathcal{G}^{-1} \mathcal{G}\partial_{p_\lambda} \mathcal{G}^{-1}\,, \qquad (6)$$

have been used in our paper with Grinevich to describe the topological protection of the Weyl fermions [104]. Here, σ is the closed 3D surface around the point in 4D momentum-frequency space. The value of this Chern number in Eqn (6) is equal to the charge of the ABH monopole in Fig. 2b. It is the instanton description of the Polyakov hedgehog-monopole in momentum space (see Section 14).

19. Larkin and disorder

One of Larkin's surprising results is that even small disorder destroys the Abrikosov vortex lattice [134]. In magnets, a similar effect is the destruction of orientational long-range order by weak random anisotropy [135]. Following this trend at the Landau Institute, it was suggested that a similar effect can be realized in ^{3}He-A in aerogel, where the weak random anisotropy provided by the disordered aerogel strands may destroy the long-range orientational order [136] (Fig. 14). This disordered state, which we call the Larkin–Imry–Ma

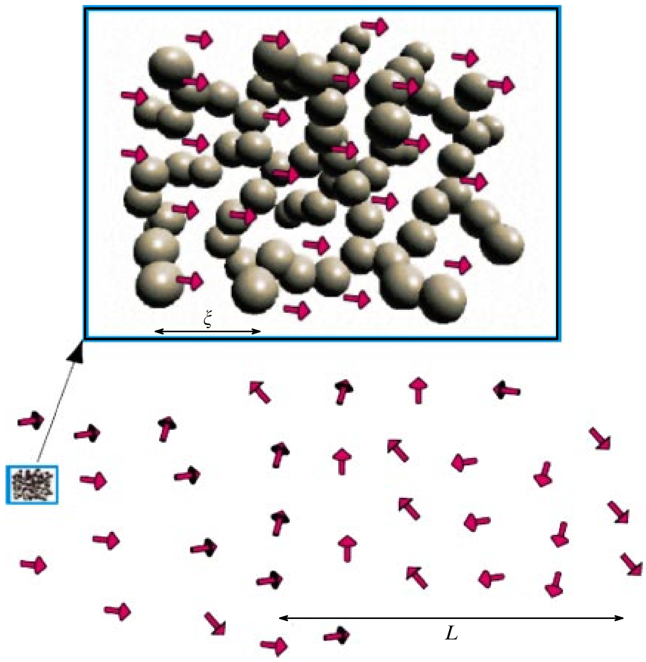

Figure 14. Larkin–Imry–Ma state of ^{3}He-A in aerogel. Long-range orientational order is destroyed by weak interaction with aerogel strands, which provide the random anisotropy. Here, L is the length scale at which the long-range order is destroyed. It is much bigger than the characteristic scale of quenched random anisotropy.

state, has been observed in NMR experiments [137, 138], which opened the route to study experimentally many different types of spin and orbital glasses in superfluid ^{3}He [98].

20. Kopnin and Iordanskii forces

Starting in 1981, I had a collaboration with an experimental team in Helsinki, which studied different types of vortices and other topological defects in a unique rotating cryostat operating at milliKelvin temperatures. For this, I had to study vortex dynamics, which at that time was being developed by Kopnin in superconductors.

At first glance, the Kopnin theory of vortex dynamics [139, 140] looked rather complicated. Fortunately, it happened that his theory could be reformulated in more simple terms. The vortex represents the chiral system, and the spectrum of the fermionic modes localized in the vortex core has an anomalous branch which, as a function of the discrete angular momentum L_z, 'crosses' zero (Fig. 15a, c). It appears that the spectral flow along this anomalous branch is responsible for the Kopnin force acting on the vortex. Thus, the Kopnin force can be explained in terms of the chiral anomaly as an analog of the Callan–Harvey effect [141]. The chirality here is generated by the vorticity: the number of anomalous branches which 'cross' zero is determined by the vortex winding number N.

The Kopnin spectral flow force adds to the conventional Magnus force acting on the vortex, which exists in conventional liquids, and to the Iordanskii force, already well known in two fluid dynamics of superfluids [142, 143]. The Kopnin spectral flow force is of fermionic origin and exists only in fermionic superfluids and in superconductors.

After the origin of the Kopnin force was clarified, our cooperation with Kopnin on vortex dynamics was developed [144, 145]. Finally, the Kopnin theory was confirmed in experiments on vortices in ^{3}He-B [30]: the measured temperature dependence of the Kopnin force agreed with his calculations. Note that bulk superfluid ^{3}He does not contain impurities, and vortices are not pinned. This allowed us to measure the Kopnin, Iordanskii, and Magnus forces in their pure form (Fig. 16).

In continuous vortices in ^{3}He-A — skyrmions — the Kopnin force can be fully described by the Adler–Bell–Jackiw equation for chiral anomaly:

$$\partial_\mu J_5^\mu = \frac{1}{4\pi^2} q^2 \mathbf{BE}\,, \tag{7}$$

which confirms the chiral anomaly origin of the Kopnin force in the general case. In Eqn (7), the synthetic gauge fields come from the time and space dependence of the position of the node $\mathbf{K}(\mathbf{r}, t)$ in the spectrum of Weyl quasiparticles in the presence of moving skyrmions: $\mathbf{B} = \nabla \times \mathbf{K}$ and $\mathbf{E} = \partial_t \mathbf{K}$. In the weak coupling approximation, the Kopnin force compensates the Magnus force practically at any temperature, which has been confirmed in experiments on vortices in ^{3}He-A [30].

The Kopnin force was also very important in the study of turbulence in the flow of superfluid ^{3}He. Since the Kopnin force has a similar dependence on velocity as the mutual friction force between the normal component of the liquid and quantized vortices, the corresponding Reynolds number $\mathrm{Re}\,(T) = \omega_0 \tau$ does not depend on velocity and is only

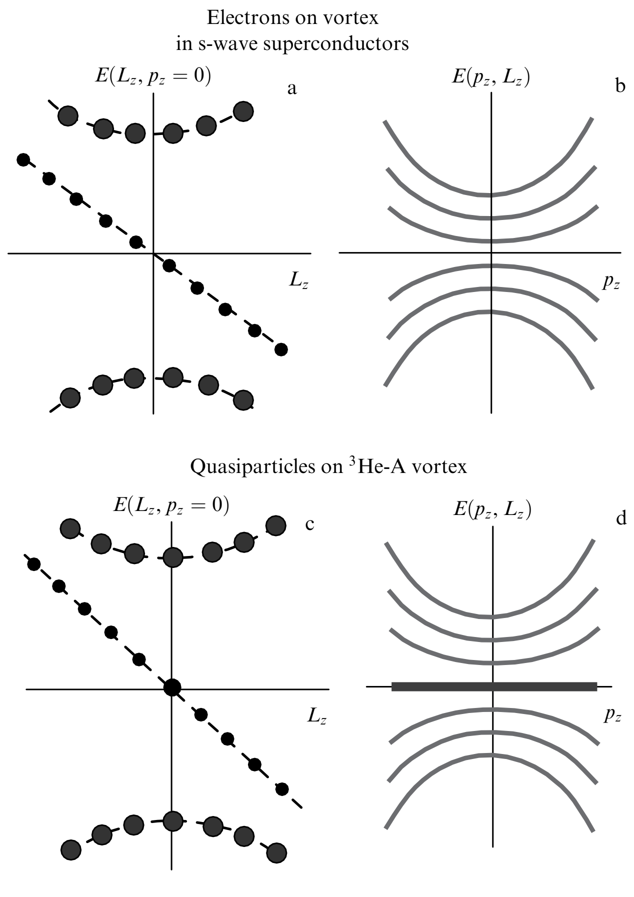

Figure 15. (a) Chiral branch of fermions existing in the core of Abrikosov vortices in s-wave superconductors. The spectrum is the function of discrete angular momentum L_z. The spectral flow along this anomalous branch is the origin of the extra force acting on a vortex — the Kopnin force. (b) The spectrum as function of p_z at fixed values of L_z. The spectrum has a minigap of the order of $\omega_0 = \Delta^2/E_F$. (c) Chiral branch of fermions existing in the core of vortices in a chiral p-wave superfluid. This spectrum contains the Majorana zero energy mode. (d) The spectrum as a function of p_z in superfluid ^{3}He-A, where the branch with $L_z = 0$ represents the flat band of Majorana fermions.

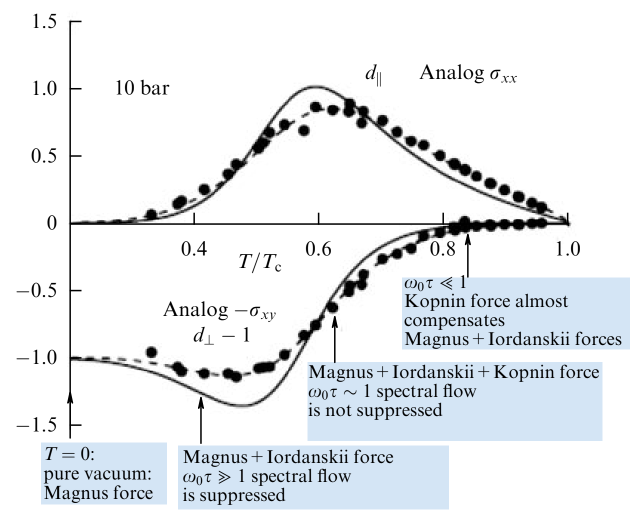

Figure 16. Measurement of three nondissipative forces acting on quantized vortices in rotating ^{3}He-B: the Magnus force, known in conventional liquids, the Iordanskii force, known in two-fluid dynamics of superfluids, and the Kopnin spectral flow force, which exists only in Fermi superfluids and comes from an analog of the chiral anomaly for fermions existing in the vortex core. The solid line is the Kopnin calculations.

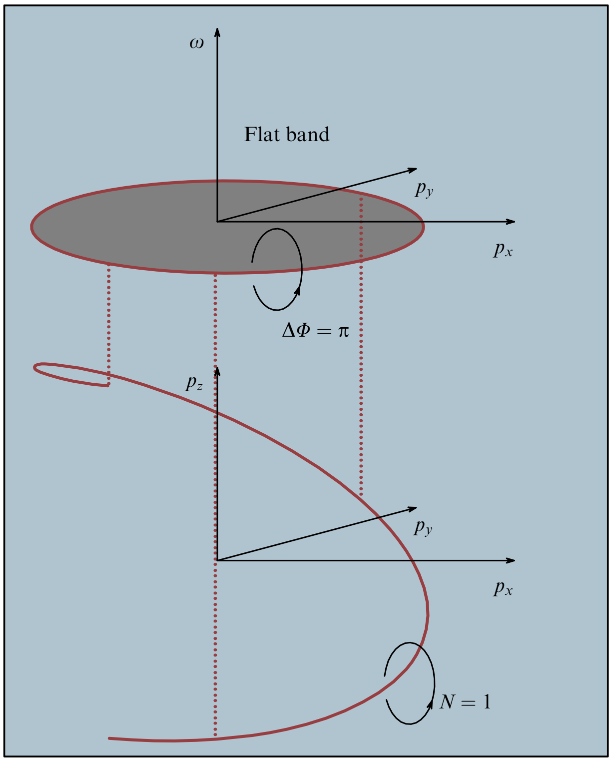

Figure 17. Bulk-surface correspondence in semimetals with the nodal line in bulk. Dirac nodal line gives rise to a flat zero-energy band on the surface. The boundary of the flat band coincides with the projection of the Dirac line to the surface.

determined by temperature, where ω_0 is the minigap and $1/\tau$ is the width of the vortex core levels. The transition from the laminar to turbulent flow takes place at the temperature when $\mathrm{Re}(T) \sim 1$. Such a transition governed by this novel Reynolds number has been experimentally observed (see the review [146]).

21. Kopnin, Majorana fermions, and flat band superconductivity

A very interesting result obtained by Kopnin concerns fermion modes in the core of the singular $N = 1$ vortex in chiral superfluid ^{3}He-A (Fig. 15c, d) [112–114]. It was found that the branch of the spectrum with zero angular momentum $L_z = 0$ is dispersionless, $E_0(p_z) = 0$ in some region of momenta, $-p_{\mathrm{F}} < p_z < p_{\mathrm{F}}$ (Fig. 15c, d). This observation inspired the search for the topological origin of this 1D flat band with zero energy. It appeared that in the 2D case the state with exactly zero energy corresponds to the Majorana mode at the vortex [114, 147]. In the 3D case, the existence of the 1D Majorana flat band is connected to the existence of the Weyl nodes in bulk: the boundaries of the flat band are determined by the projections of the Weyl nodes to the vortex line in bulk [148]. This is one of the many examples of bulk-boundary and bulk-vortex correspondence in topological materials.

An even more important example is the 2D topological flat band on the surface of semimetals having nodal lines in the bulk spectrum. The boundary of the surface flat band is determined by the projection of the nodal line to the surface of the semimetal [149] (see Fig. 17). The singular density of states in the flat band leads to flat band superconductivity [150–152]. The flat band superconductivity is characterized by the linear dependence of the transition temperature on the interaction g in the Cooper pair channel, $T_{\mathrm{c}} \sim g V_{\mathrm{FB}}$, where V_{FB} is the volume or the area of the flat band [153]. This is in contrast to conventional superconductvity in metals with Fermi surfaces, where T_{c} is exponentially suppressed.

Recently, superconductivity has been observed in twisted bilayer graphene [156, 157]. The maximum of T_{c} takes place at the 'magic angle' of the twist, at which the electronic band structure becomes nearly flat (see discussion in Refs [156, 157]) (Fig. 18).

For vortices in superfluids and superconductors with nodal lines in bulk, the singularities in the thermodynamics come from regions far away from the vortex (see the discussion on 'koreshok' in Section 16). For cuprate superconductors, they are discussed in [127] and in the paper by Kopnin and me [158].

22. Fomin, coherent precession, magnon BEC

Fomin received the London Prize together with the experimentalists from the Kapitza Institute, Bunkov and Dmitriev, for the discovery of the spontaneously formed coherent precession of magnetization in superfluid ^{3}He-B [159–161]

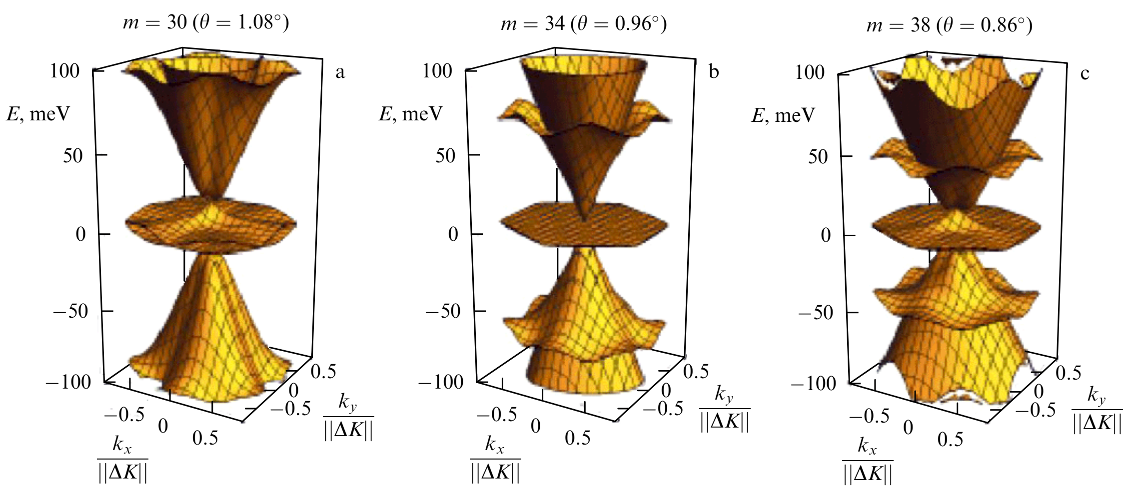

Figure 18. Spectrum of electrons in twisted bilayer graphene, where the flat band emerges at the magic angle of the twist (from Ref. [157]).

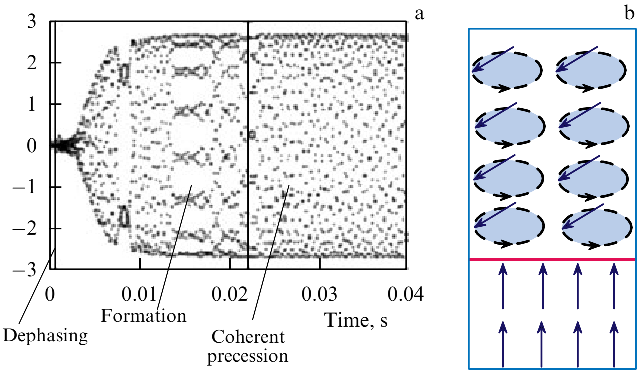

Figure 19. Spontaneously formed coherent precession of magnetization in ^{3}He-B discovered at the Kapitza Institute in collaboration with Fomin. (a) After a strong pulse of a radio-frequency field, which deflects magnetization at a large angle, the spins start to process around the external magnetic field. Due to inhomogeneity of the system, the spins precess at different frequences, which leads to dephasing, and the measured signal completely disappears. Then a miracle occurs: without any external influence, a coherent dynamical state is spontaneously formed, in which all the spins precess with the same collective frequency and with the same phase, ignoring the inhomogeneity of the system. The precessing state is concentrated in the part of the cell (b) called the Homogeneously Precessing Domain (HPD). The precession slowly decays, but during the decay it remains phase coherent, only the volume of the precessing domain slowly decreasing with time.

(Fig. 19). This is a unique example of spontaneous self-organization in a quantum system. Spins which originally processed with different frequencies form the collective state in which all the spins precess at the same frequency and with the same phase. This state lives for a long time without external influence and in spite of the inhomogeneity of the system.

For me, this phenomenon was not very clear. However, it worked. Due to the connection between the Kapitza Institute and the Low Temperature Lab in Helsinki, this phenomenon was brought to Helsinki, where the HPD appeared to be very useful as a tool for experimental investigation of topological defects in ^{3}He-B — vortices and solitons. In particular, using HPD, an exotic topological object — combined spin-mass vortex with a soliton tail — has been observed and identified [162]. So, I had to study this phenomenon in detail.

Again, as in the case with Kopnin's theory of vortex dynamics, the Fomin theory of HPD was beautiful, but it was very difficult for me to apply it to our problems. And again, it happened that Fomin's theory could be reformulated in a more simple way: in terms of the Bose–Einstein condensate (BEC) of quasiparticles — magnons [163] (see also review [164]). The reason for that is that coherent precession has an off-diagonal long-range order (ODLRO) signature similar to that in a conventional superfluid:

$$S_x + \mathrm{i}S_y = \langle \hat{S}^+ \rangle = S \sin\beta \exp\left[\mathrm{i}(\alpha + \omega t)\right], \tag{8}$$

where $\hat{S}^+$ is the operator of creation of spin and β is the tipping angle of magnetization. Using the Holstein–Primakoff transformation, one can rewrite this in terms of the BEC magnon, where the order parameter is the quasi-average of the operator of annihilation of the magnon number:

$$\Psi = \langle \hat{\Psi} \rangle = \sqrt{\frac{2S}{\hbar}} \sin\frac{\beta}{2} \exp\left[\mathrm{i}(\alpha + \mu t)\right]. \tag{9}$$

The role of the global phase of precession ω is played by the chemical potential μ of the pumped magnons.

The close connection between the coherent precession of spins and superfluid/superconducting states with off-diagonal long-range order is also supported by the observation that superconductivity can be represented as the coherent precession of Anderson pseudospins [165]. However, in contrast to long-lived, but quasi-equilibrium coherent precession of ordinary spins, superconductivity is a phenomenon of true equilibrium. The reason for this is that the projection of the total Anderson pseudospin coincides with the number of electrons and, therefore, is completely conserved, in contrast to the quasiconservation of the number of magnons.

As a result, neglecting the spin relaxation, the dynamics of the precessing system can be determined by the corresponding Landau–Khalatnikov hydrodynamics, applied now to a magnon superfluid. Then, all the phenomena related to coherent precession, old and new, could be described in the same way: the spin current Josephson effect; the magnon BEC in magnetic and textural traps; the Goldstone mode of precession — the phonon in magnon BEC; the magnonic analog of MIT bag model of hadrons [166]; the magnonic analog of the relativistic Q-ball [167]; etc.

The coherent precession also has some signatures of the so-called time crystal [168]. If the spin-orbit interaction is ignored, the magnon number is conserved, and the precessing state is the ground state of the system with a fixed number of magnons. So, we have the oscillations in the ground state, as suggested by Wilczek [169]. But without spin-orbit interaction, these oscillations are not observable.

A theory of magnon BEC in solid state materials (yttrium iron garnet films) was considered by Pokrovsky (see review [170]).

23. Polyakov, Starobinsky, the cosmological constant, and vacuum decay

The counterpart of the Polyakov hedgehog-monopole in momentum space — the Weyl point — naturally gives rise to the emergent gravitational field acting on Weyl fermions. This again sparked my interest in topics related to gravity, but now on more serious grounds than the analogy with superfluid ^{4}He in Section 1. In this respect, the consultations with Starobinsky became highly important and extremely useful.

One of the areas was black hole radiation, which was started by Zel'dovich [171] and Starobinsky [172] for rotating black holes and continued by Hawking for nonrotating black holes. At the Landau Institute, this issue was rather popular: I can mention Belinski [173], with whom I had many discussions, and Byalko [174].

It appeared that Zel'dovich–Starobinsky radiation by a rotating black hole can be simulated by a body rotating in a superfluid vacuum [175, 176], while Hawking radiation can be simulated using superflow or a moving texture [7, 8]. Also, both Hawking radiation and Zel'dovich–Starobinsky radiation can be described in terms of semiclassical tunneling. The same semiclassical approach can be applied to the radiation from the de Sitter cosmological horizon. However, this now touches a different area — problems related to the vacuum energy and cosmological constant. In this area, Starobinsky inflation [177, 178] is the key issue.

In a series of papers by Klinkhamer and me [179–181], we introduced the so-called q-theory, where the vacuum is described by a dynamical variable introduced by Hawking, the 4-form field [182–184]. The nonlinear extension of the Hawking theory allowed us to study the thermodynamics and dynamics of the quantum vacuum. The approach appeared to be rather general. Instead of the Hawking 4-form field, one may use other variables which can describe the physical vacuum, but they lead to the same dynamical equations. One such variable [185] was inspired by the papers by Kats and Lebedev on a freely suspended film [186].

The main advantage of such an approach is that, in full equilibrium, the properly defined vacuum energy, which enters the Einstein equations as the cosmological constant, is zero without fine tuning. The mechanism of cancellation is purely thermodynamic and does not depend on whether the vacuum is relativisic or not. In this respect, it is very different from the Pauli-Zeldovich mechanism discussed by Kamenshchik and Starobinsky [121], which relies on the cancellation of contributions of relativistic bosons and relativistic fermions.

The thermodynamic approach solves the main cosmological constant problem: in a Minkowski vacuum, the huge vacuum energy is naturally cancelled. The problem remains, however, in the dynamics. If one assumes that the Big Bang started in an originally equilibrium vacuum, then from our equations without dissipation it follows that the cosmological constant, which is very large immediately after the Big Bang, relaxes with oscillations and its magnitude averaged over fast oscillations reaches the present value in the present time [180] (Fig. 20):

$$\langle \Lambda(t_{\text{present}}) \rangle \sim \frac{E^2_{\text{Planck}}}{t^2_{\text{present}}} \sim 10^{-120} E^4_{\text{Planck}} \,.$$

This process looks similar to the Starobinsky inflation, except for the magnitude of the oscillation frequency, which in our case is on the Planck scale instead of the Higgs inflaton mass.

A similar oscillating decay takes place in superconductors after quenching [187–189] (see Fig. 20, bottom). Such oscillations, with the frequency equal to the mass (gap) of the Higgs amplitude mode, $\omega = 2\Delta$, have been observed experimentally [190, 191]. In superfluids and superconductors the role of the vacuum energy is played by $(\Delta^2(t) - \Delta_0^2)^2$ (see Section 7.3.6 in [12]). Then, one has

$$\Lambda(t) \propto \omega^3 \frac{\sin^2(\omega t)}{t} \,, \quad \langle \Lambda(t) \rangle \propto \frac{\omega^3}{t} \,.$$

But, if the initial conditions are different, then from our equations (again still without dissipation) it follows that the Universe relaxes to the de Sitter spacetime instead of the Minkowski vacuum state. The questions arise: what is the fate of the de Sitter vacuum? Does Hawking radiation from the de Sitter cosmological horizon exist? If the de Sitter vacuum radiates, does the Hawking radiation lead to a decrease in the vacuum energy? Is the de Sitter vacuum unstable? The instability of the de Sitter vacuum is supported by Polyakov [192–195], but is not supported by Starobinsky. My view on that problem is in papers [196, 197], which is closer to the Starobinsky view.

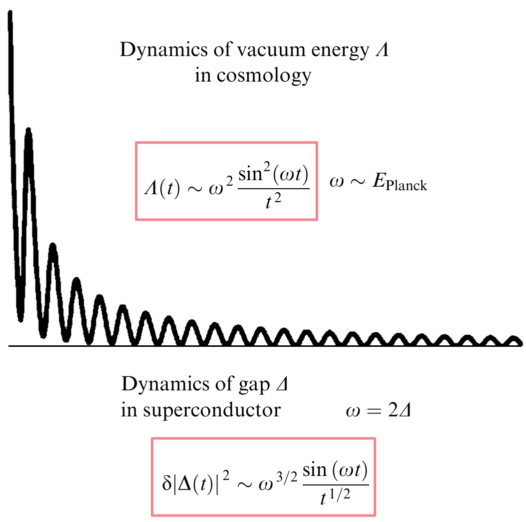

Figure 20. Processes of vacuum decay after the Big Bang in the Universe and after quenching in superconductors and fermionic superfluids. In both cases, the decay is accompanied by oscillations with frequency corresponding to the mass of the inflaton — the Higgs field or the q-field.

24. Conclusion

The overwhelming majority of my work emerges from the Landau Institute environment, and/or in collaboration with the experimental ROTA group in the Low Temperature Laboratory of Aalto University. I did not mention here the inspiration from and/or the direct collaboration with Edel'stein [198], Eliashberg, Kats [199], Khmel'nitskii [200], Makhlin [79, 196, 197, 201], Mel'nikov [198], Pokrovsky, Rashba, Sinai, and other colleagues from the Landau Institute in different areas of physics, who were also gathered by Khalatnikov into this unique institute.

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